Discussion of Cantilever Spring Stiffness Coefficient in Inertia Experiment

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Abstract: In the inertia experiment, the spring stiffness coefficient of the cantilever has been regarded as a constant. When changing the mass of the calibration block, whether the spring stiffness coefficient k of the cantilever will affect the experimental results is not clear. In this paper, the spring stiffness coefficient and the inverse relationship between the spring stiffness coefficient and the mass of the cantilever are used to show that the spring stiffness coefficient k of the cantilever is not affected by the mass, so it can be treated as a constant.

Introduction

In the experimental teaching, the analysis of the experimental results of the error source is an important step to assess the experimental results. In the inertial of the experiment, by measuring the vibration cycle of the object to its quality, and the vibration period T^2 is proportional to m_i. In this result, we do not consider the effect of the cantilever spring stiffness coefficient k on the experimental results, and whether the cantilever spring stiffness coefficient k has an effect on the experimental results, we will focus on the study in this paper.

In 2001, Liu et al. used inertia as an example to analyze the causes of mapping errors, and pointed out that the smaller the coordinate scale value, the mapping error will be reduced[1]. In 2003, Zhang published “Reflection on the ‘Inertia Scale’ Experiment” on “Physical Experiment”, which introduced the basic principles of the inertia scale and analyzed the results of testing the inertia scale at different placements [2]. In 2004, Tang et al. analyzed the influence of gravity on the experiment of inertia, the relationship between gravity and inertia, and the periodicity formula given in different cases, and gives the corresponding experimental argument [3]. In the course of the experiment, when the quality changes, the cantilever's spring stiffness coefficient k will affect the experimental results but no clear statement.

Measurement of Inertia Quality

As we all know, as long as an object has quality, it has inertia, and inertia quality is measurement for the object to maintain the original state of motion. The inertia scale is a simple experimental device used to measure the inertia mass of an object. The inertia is calculated by measuring the vibration cycle of the motion of the object to calculate the magnitude of the inertial mass. When the inertia is said to be horizontally placed, the inertia is the following relationship between the cantilever vibration period and the inertial mass:

$$T^2 = \frac{4\pi^2}{k} m_0 + \frac{4\pi^2}{k} m_i \tag{1}$$

Where T is the vibration period of the object, k is the spring stiffness coefficient of the cantilever, m0 is the weight of the unloaded mass, and the inertia mass of the measurement object mi. In this experiment, we see k as a constant, and in fact, we are not sure whether k will change with the quality of the experimental results lead to greater error. This article will address this issue.

Experimental Measurement and Data Recording

The inertia scale is horizontally fixed and flattened, and the quality of the calibration block is measured by the electronic balance. The quality of the calibration block is based on the actual measurement. From the no-load will be a block of a block added to the inertia, respectively, recorded different quality of the 30 vibration cycles, until the calibration block all added to the inertia so far. And then take a block to repeat the above measurements to no load. The above steps were repeated four times, averaged over 30 cycles t of the measured vibration, and the square processing of T and T was obtained, and the data was recorded in Table 1.

Table 1 Calibration Data when the Inertia Scale is Placed Horizontally T= t/30.

m /kg	t_1 /s	t_2 /s	t_3 /s	t_4 /s	t_5 /s	t_6 /s	t_7 /s	t_8 /s	$\overline{T}$/s	T/s	T^^{2}/s
0.0000	8.88	9.00	8.93	9.02	8.90	8.99	8.89	9.02	8.954	0.298	0.089
0.0251	10.62	10.68	10.75	10.68	10.63	10.66	10.67	10.70	10.674	0.356	0.127
0.0500	12.16	12.20	12.21	12.17	12.18	12.16	12.19	12.15	12.178	0.406	0.165
0.0750	13.49	13.51	13.49	13.50	13.48	13.50	13.50	13.49	13.495	0.450	0.202
0.1038	14.74	14.77	14.75	14.76	14.75	14.75	14.76	14.75	14.754	0.492	0.242
0.1239	15.92	15.94	15.97	15.96	15.95	15.92	15.93	15.93	15.940	0.531	0.282
0.1491	17.05	17.03	17.01	17.01	17.04	17.05	17.04	17.03	17.033	0.568	0.322
0.1745	18.11	18.09	18.08	18.07	18.10	18.11	18.10	18.11	18.096	0.603	0.364
0.2003	19.16	19.13	19.19	19.12	19.18	19.15	19.14	19.16	19.154	0.638	0.408
0.2254	20.16	20.11	20.15	20.09	20.14	20.11	20.13	20.15	20.130	0.671	0.450
0.2501	21.12	21.11	21.18	21.08	21.10	21.13	21.14	21.12	21.123	0.704	0.496

Experimental Data Processing. We know that when an object is not affected by gravity,

$$T = 2\pi\sqrt{\frac{m_0 + m_i}{k}} \qquad (2)$$

When the inertia scale is placed horizontally, the influence of gravity is not taken into account. The relationship between the vibration period T^2 of the inertia scale not affected by gravity and the mass m_i of the object to be measured is linear. Make a T^2-m_i calibration line. The cantilever spring stiffness coefficient k and the no-load mass m_0 can be obtained by the slope and intercept of the calibration line. As long as we measure the vibration period of the object to be measured, the T^2-m_i calibration line can be determined by the quality of the object to be measured.

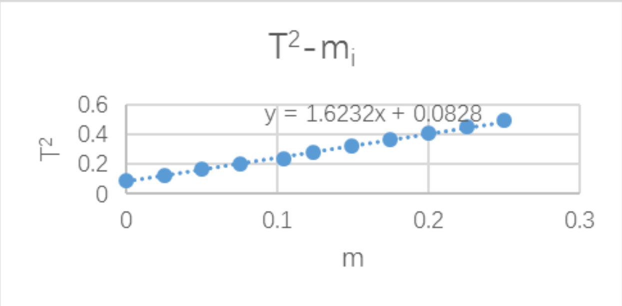

Fig. 1. T^2-m_i linear fit graph

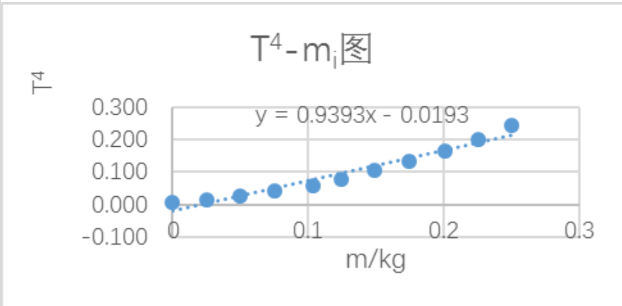

Fig. 2. T^4-m_i linear fit graph

The T2 and mi are plotted in the coordinate system and linearly fitted. As shown in Fig. 1, the spring stiffness coefficient k and the equivalent mass m0 of the vibrator are calculated by fitting the parameters.

It can be known from formula (2) that:

$$T_i^2 = \frac{4\pi^2}{k}(m_0 + m_i) \qquad (3)$$

Put the data into the formula to obtain:

$$k = 24.32N/m, m_0 = 51.04g \qquad (4)$$

In the above calculation, the default spring stiffness coefficient k is a constant. In order to study whether we change the mass spring stiffness factor, we assume that the spring stiffness coefficient of the cantilever is proportional to the mass, and then set:

$$k = AT^2 \tag{5}$$

In which A is a constant, and put it into formula (1) to obtain:

$$\begin{aligned} T^2 &= 4\pi^2\left(\frac{m_0+m_i}{k}\right) \\ &= 4\pi^2\left(\frac{m_0+m_i}{AT^2}\right) \end{aligned} \tag{6}$$

Square both sides of formula (6) to obtain:

$$T^4 = \frac{4\pi^2}{A}(m_0+m_i) \tag{7}$$

If T^4= y, m_i = x and then:

$$y = \frac{4\pi^2}{A}x + \frac{4\pi^2}{A}m_0 \tag{8}$$

It can be seen from Fig. 2 that the linear fitting is not ideal and the constant A can be obtained from the slope of the straight line according to the fitting data, i.e.:

$$A = \frac{4\pi^2}{0.9393} = 42.03 \tag{9}$$

$$m_0 = -20.55g \tag{10}$$

As the inertia of the table called no-load quality cannot be negative, so this assumption is wrong. It can be seen that the inertia of the cantilever spring stiffness coefficient and the quality is not a simple proportional relationship between. Therefore, we again speculate that the cantilever spring stiffness coefficient is inversely proportional to the quality, and if:

$$k = \frac{B}{m_0+m_i} \tag{11}$$

In which B is the constant, and it can be obtained from the above formula that:

$$T^2 = \frac{4\pi^2}{B}(m_0+m_i)^2 \tag{12}$$

If T^2 = y , m_i = x , and then

$$y = \frac{4\pi^2}{B}\left(x^2+2m_0x+m_0^2\right) \tag{13}$$

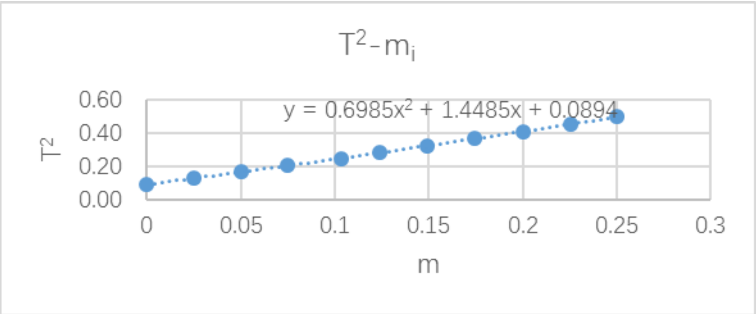

Fig. 3. T^4-m quadratic fit graph

B and m0 can be obtained according to the above fitting data, and the specifics are as follows:

$$B = \frac{4\pi^2}{0.6985} = 56.52 \tag{14}$$

$$m_0 = 35.78g \tag{15}$$

Although the results are positive, the difference between the actual no-load quality of the inertia scale is big and ever greater than the error of the results when the spring stiffness coefficient is regarded as a constant. It is therefore certain that the relationship between the spring stiffness coefficient and the mass of the cantilever is not a simple inverse relationship, and of course it is not a linear relationship.

Summary

The above results show that the spring stiffness coefficient of the cantilever is regarded as a constant in the inertia experiment. It is one of the simplest and less wrong methods. There is no linear relationship between the cantilever spring stiffness coefficient and the mass. There may also be an inverse ratio between the two Relationship, but it is not a simple inverse relationship. The conclusion from the other side to prove that the cantilever spring stiffness factor is the physical properties of the cantilever, subject to external factors less affected.

It can be seen that the effect of cantilever spring stiffness coefficient on the experimental results is more reasonable, the results are closer to the real value, the experimental results are less error. Once again, it is proved that the cantilever spring stiffness coefficient is regarded as a constant, which not only makes the experimental results simple process, but also will reduce the measurement results of the error.

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